The Physics of Energy

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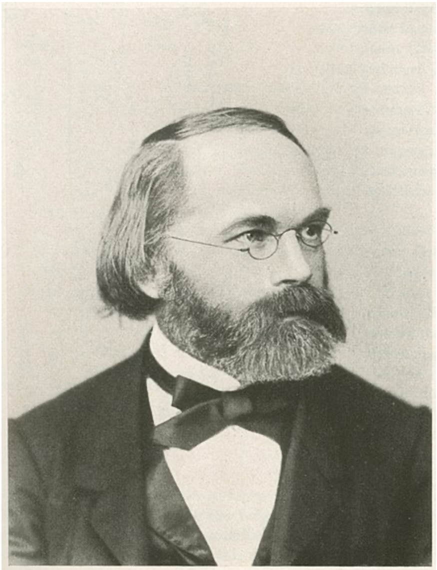
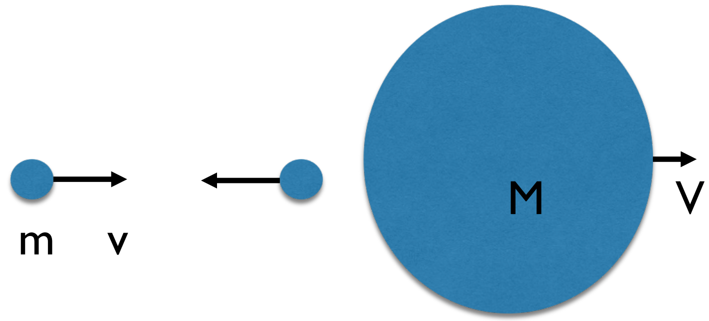
Peculiarities in the treatment of stochastic processes: the Brownian motion misinterpreted

Before Einstein 1905 model of the Brownian motion, other attempts have been made, but the atomistic interpretation was erroneously discarded.

1879 Carl Wilhelm von Nägeli (26 March 1817 – 10 May 1891) Swiss botanist





Momentum acquired by the pollen grain: *M V = 2 mv*

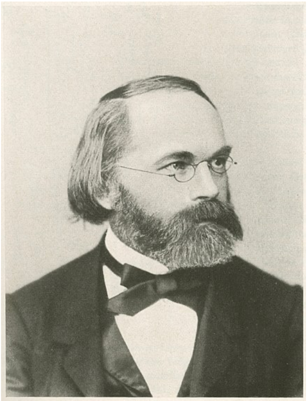
$$V = 2\,\frac{m}{M}\,v \quad \text{but} \quad \frac{m}{M} = \left(\frac{r}{R}\right)^3 = \left(\frac{5\;10^{-10}}{10^{-6}}\right)^3 \sim 10^{-10}$$

thus $V \sim 10^{-10} v$ Due to the equipartition, we have $v = \sqrt{3\,\frac{KT}{m}} \sim 6\;10^{2}\;\; m/s$

and thus $V \sim 6\;10^{-8}\;\; m/s$

This is roughly 2 orders of magnitude smaller than the observed velocity!

The stochastic force is not the sum of independent kicks



On the base of a significant disagreemnt with the observations, von Nägeli concluded that the kinetic theory model did not work.

The stochastic force is often **misrepresented** as the sum of independent kicks, half favourable, half unfavourable. This is not the case.
We should think at it as the results of a large number N kicks that happen in a very short time. The effectiveness of this force goes roughly as the sqrt(N), where N is the number of kicks per second.

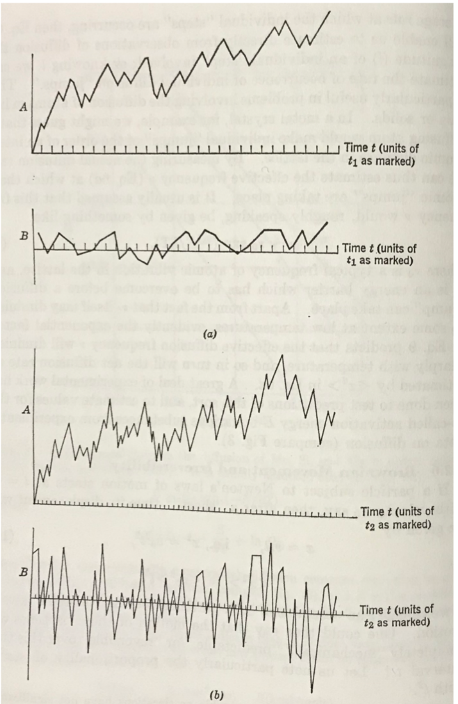
This is the result of a «vision» that takes into account a time scale separation: fast (molecular motion) vs (slow) the pollen grain motion.

Another common mistake: mind the observation time

If we calculate the standard deviation of the velocity of the pollen grain using the kinetic theory we have:

$\frac{1}{2}M\langle V^2\rangle = \frac{3}{2}KT$ which gives $\sqrt{\langle V^2\rangle} \sim 2\ 10^{-3}$ m/s orders of magnitude larger than the observed one.

Why ?



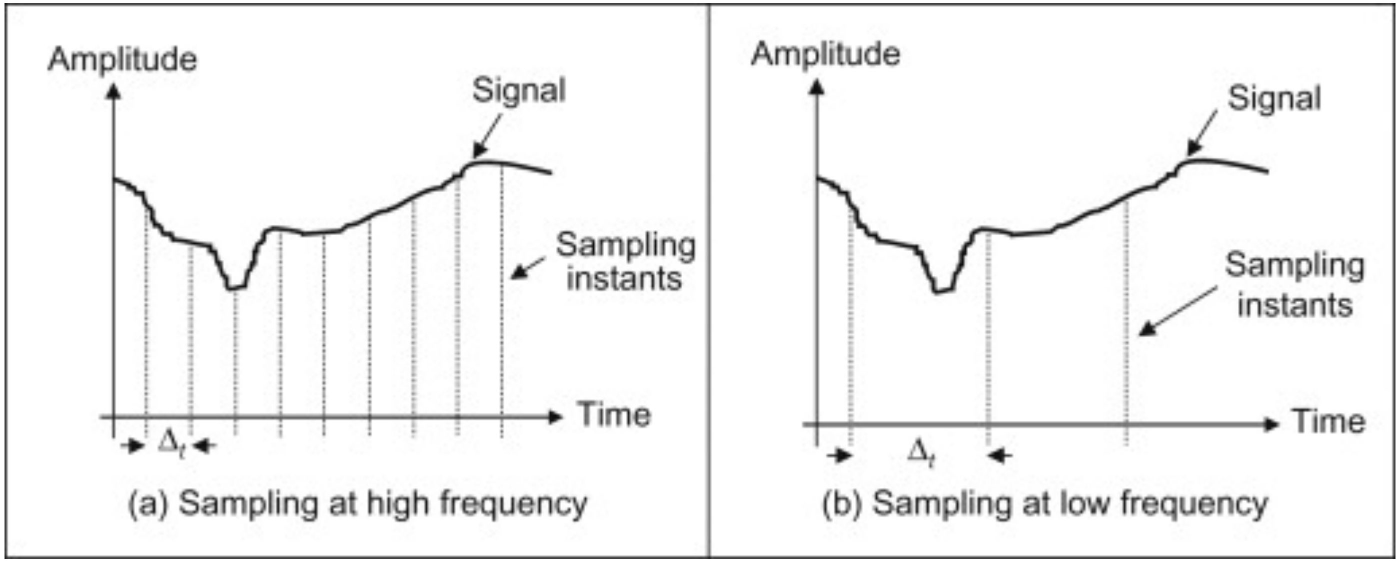
Apparently the amplitude of the velocity in this process depends on the sampling time.
Why?

The Nyquist–Shannon sampling theorem

Is a theorem in the field of digital signal processing which serves as a fundamental bridge between continuous-time signals and discrete-time signals. It establishes a sufficient condition for a sample rate that permits a discrete sequence of samples to capture all the information from a continuous-time signal of finite bandwidth.

Time discretization

If we have a continuous signal x(t) we can represent it by means of a series of time discret samples, each separated by a time distance ΔT: $x(t0 + i\,\Delta T)$ for 1= 1…n.



The Nyquist–Shannon sampling theorem

If a function x(t) contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced 1/(2B) seconds apart.

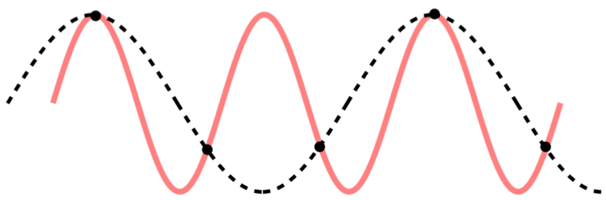
$$\Delta T < \frac{1}{2B}$$

From the sampled values we can reconstruct the continuous function as follows:

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT) \cdot \mathrm{sinc}\left(\frac{t-nT}{T}\right) \qquad \text{with} \qquad \mathrm{sinc}(x) = \frac{\sin x}{x}$$

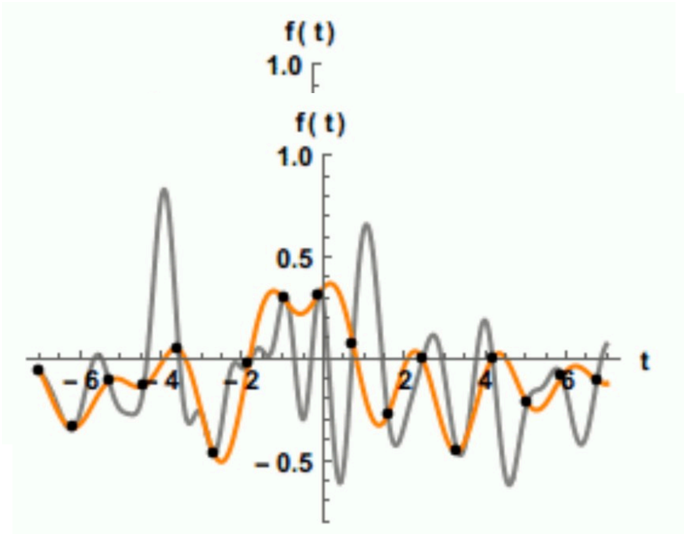
Whittaker–Shannon interpolation formula

If we do not respect the prescription of the Nyquist–Shannon sampling theorem, we have an error called **aliasing**.



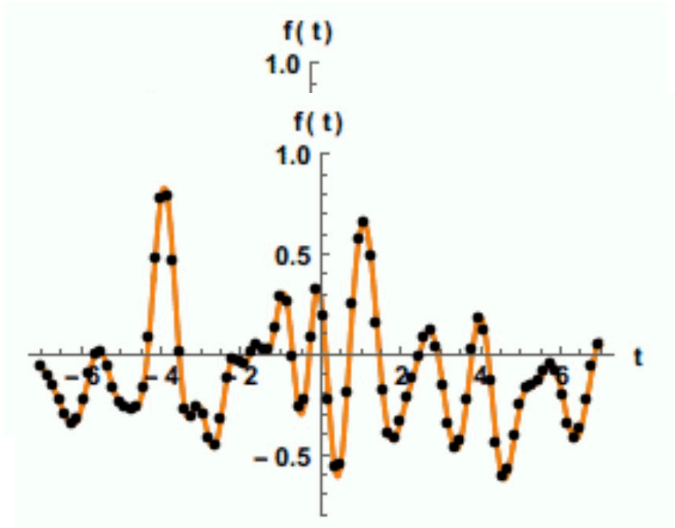
The reconstructed signal is different from the original one and, specifically has amplitude in lower frequency regions with respect to the original.

Aliasing error



Undersampled signal

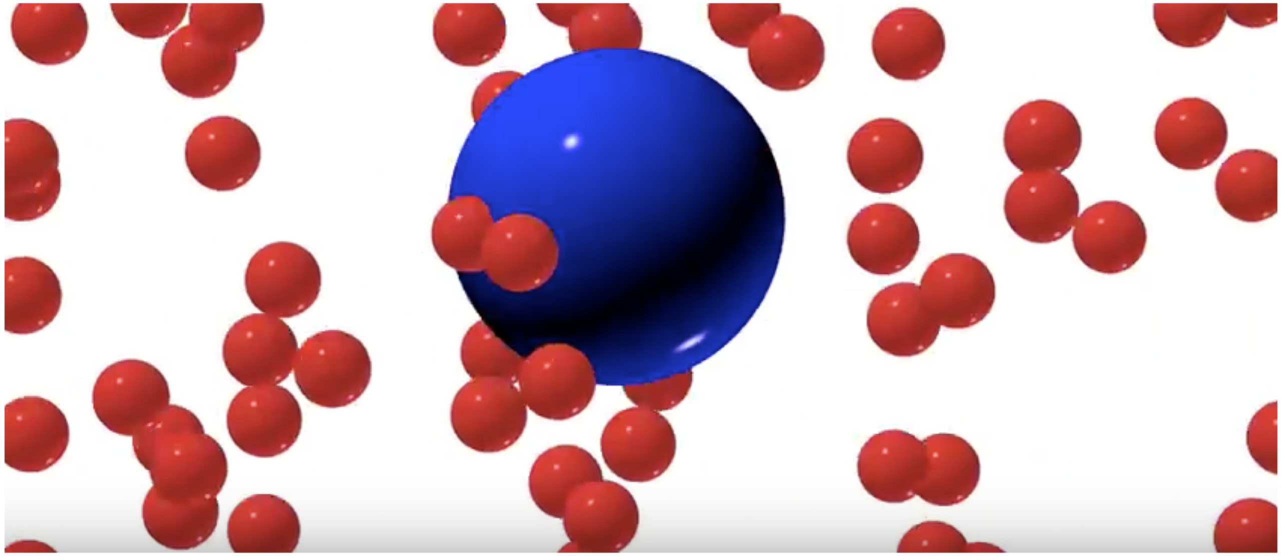
$$\Delta T > \frac{1}{2B}$$

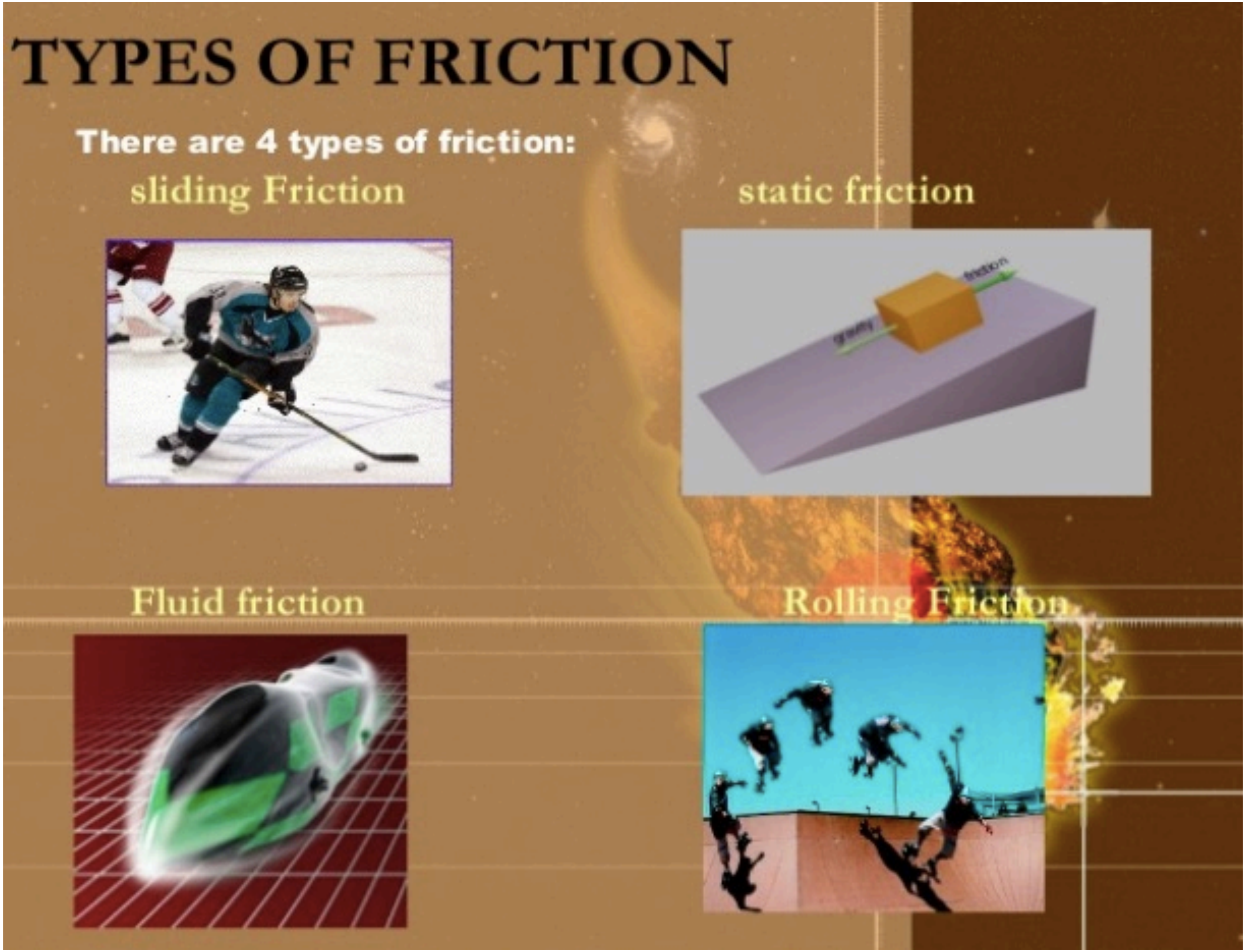


Properly sampled signal

$$\Delta T < \frac{1}{2B}$$

Friction




TYPES OF FRICTION
There are 4 types of friction:
sliding Friction
static friction
Fluid friction
Rolling Friction

What is friction?

Friction is the process by which energy contained in few degrees of freedom is transferred to many degrees of freedom.

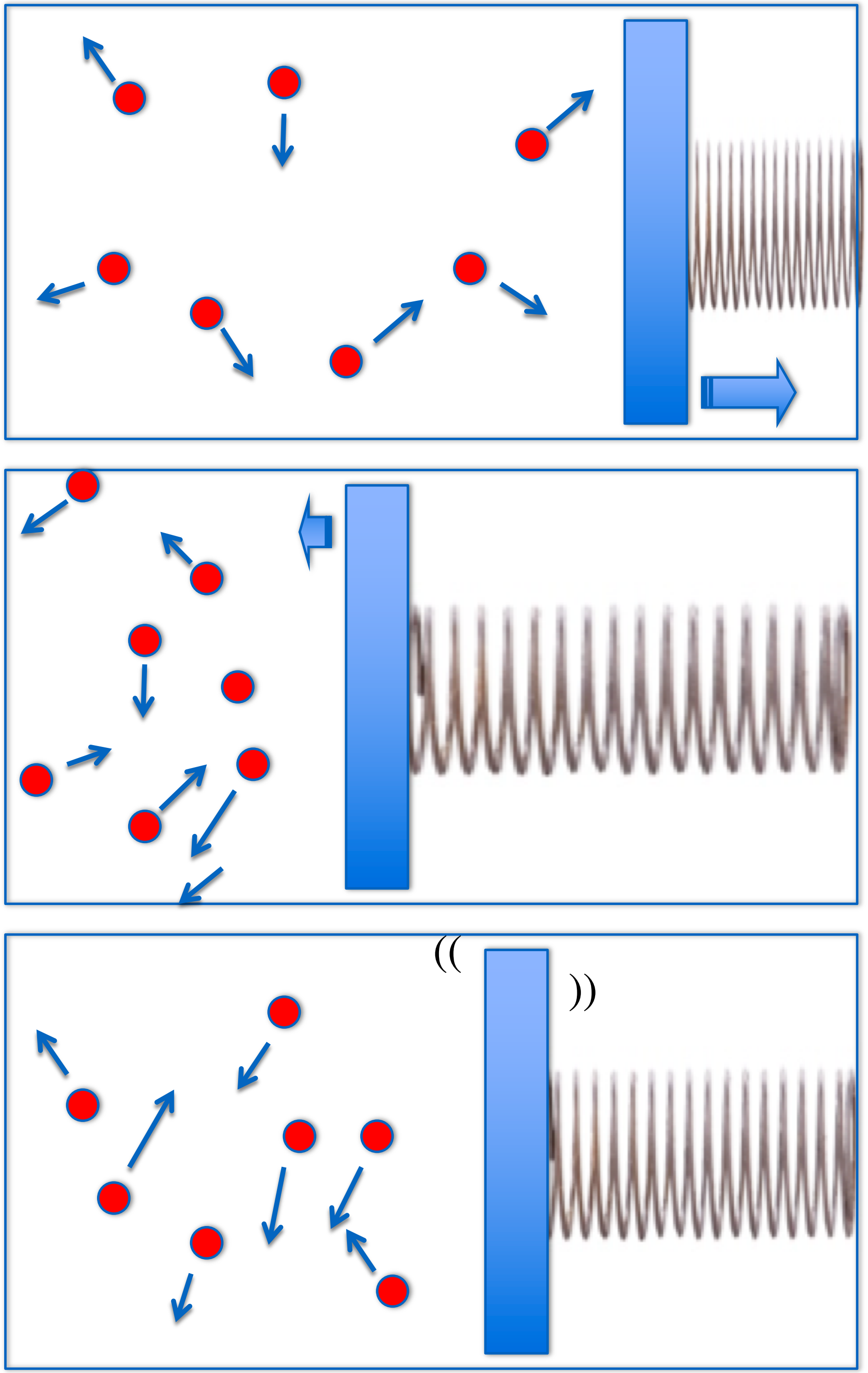
Let’s see an example...

First case

We compress the spring to some extent and then we release the compression leaving it free to oscillate.

After few oscillations we observe that the oscillation amplitude decreases as a consequence of what we call the friction (viscous damping force) action due to the presence of the gas.
The decrease ceases when the oscillation amplitude reaches a certain equilibrium value and after that it remains constant (on average).

Some energy has been dissipated into heat.

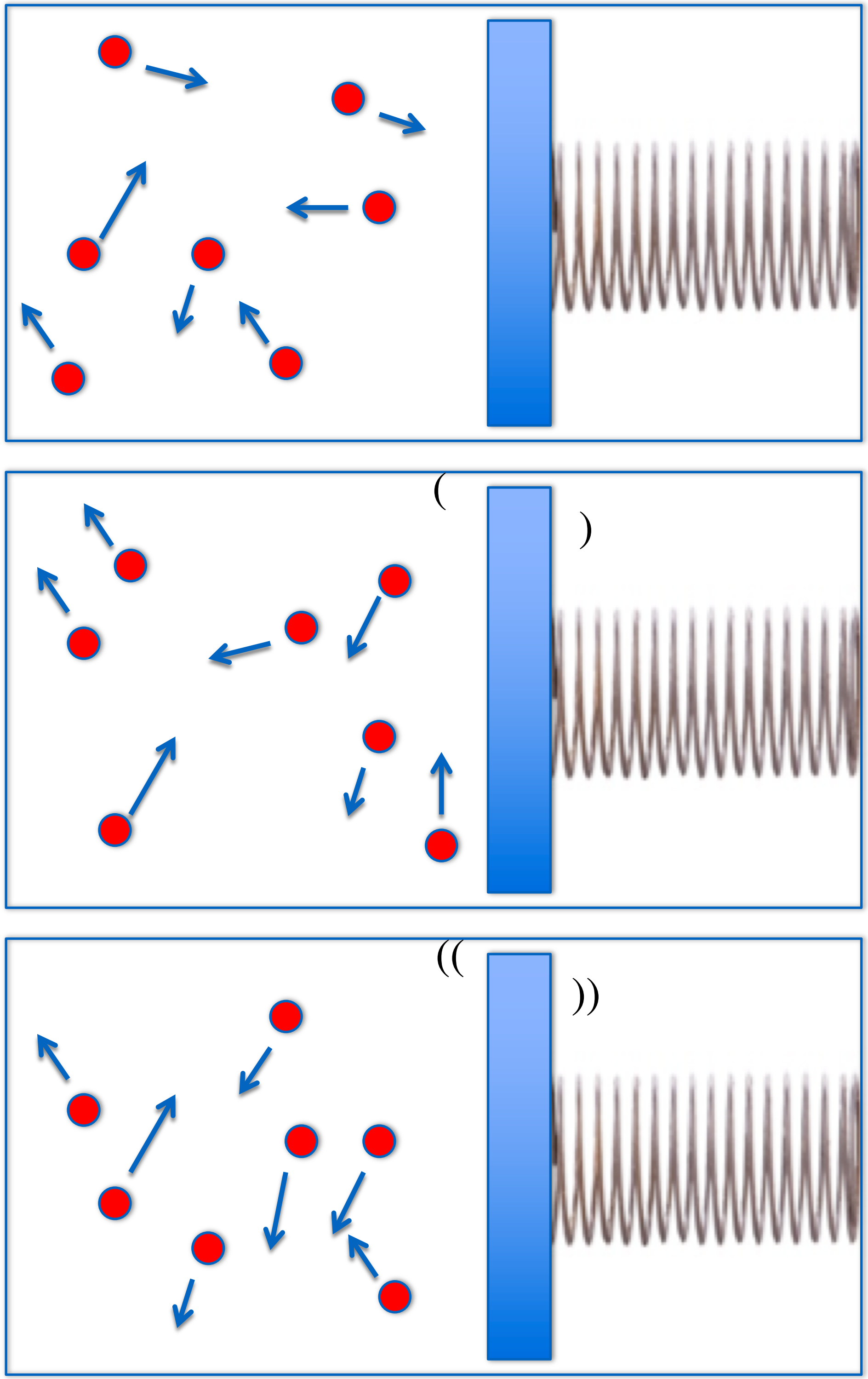


Second case

We now start with the movable set at rest and leave it free.

After few seconds we will see that the set starts to move with increasing oscillation amplitude that soon reaches **an equilibrium condition** at the very same value (on average) of the first case.

In both cases the two different roles of damping-force and pushing-force has been played by the gas.
This fact led to think that there must be a **connection between the process of dissipating energy** (a typical irreversible, i.e. non-equilibrium process) and the process of **fluctuating at equilibrium** with the gas.



Fluctuation-Dissipation connection

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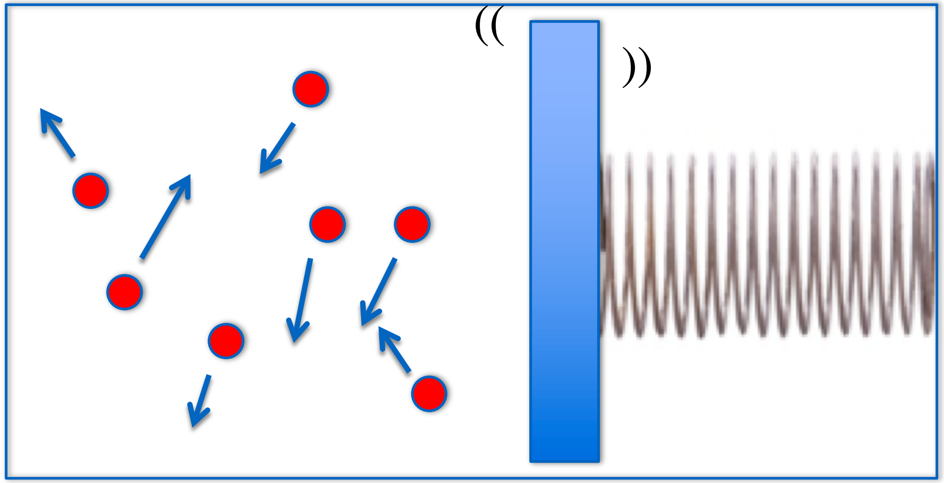


In order to unveil such a link we need to introduce a more formal description of the dynamics of the movable set.

This problem has been addressed and solved by Albert Einstein (1879 - 1955) in his 1905 discussion of the Brownian motion and subsequently by **Paul Langevin** (1872 - 1946) who proposed the following equation:

$$m\ddot{x} = -m\gamma\dot{x} - \frac{dU}{dx} + \xi(t)$$

Fluctuation-Dissipation connection

$$m\ddot{x} = -m\gamma\dot{x} - \frac{dU}{dx} + \xi(t)$$ Langevin eq.

$\xi(t)$ is the random force that accounts for the incessant impact of the gas particles on the set, assumed with zero mean, Gaussian distributed and with a flat spectrum or, delta-correlated in time (white noise assumption):

$$\langle \xi(t_1)\xi(t_2) \rangle = 2\pi\, G_R \delta(t_1 - t_2)$$

G_R accounts for the fluctuation intensity. There must be a connection with the dissipation γ.

This relation has been established within the linear response theory (that satisfies the equipartition of the energy among all the degrees of freedom) initially by **Harry Theodor Nyquist** (1889 - 1976) in 1928, and demonstrated by **Callen and Welton** in 1951.

$$G_R = \frac{mK_BT}{\pi}\gamma$$

Why is FDT important?

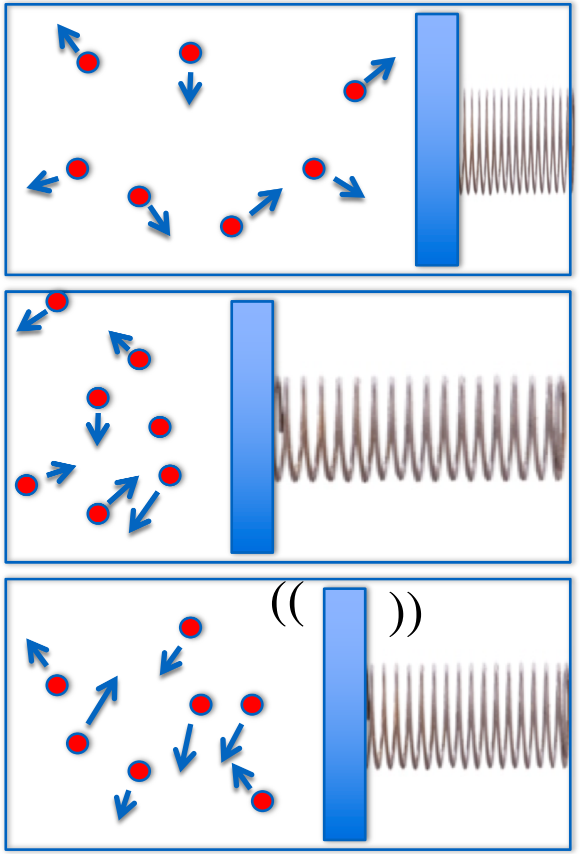
It is important because it represent an ideal bridge that connects:

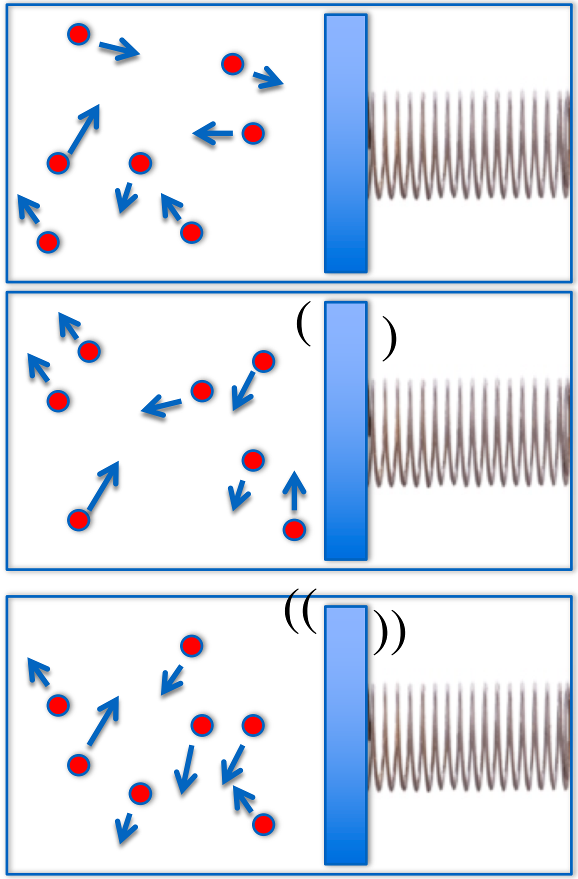
the equilibrium properties of our thermodynamic system
(represented by the amplitude and character of the fluctuations)

with

the non-equilibrium properties
(represented here by the dissipative phenomena due to the presence of the friction).

ENTROPY production – FLUCTUATION CONNECTION





dissipative properties = the capacity to produce entropy, are intrinsically connected to the equilibrium fluctuations.

Summary

- All matter is made by particles
- We can say that the energy is associated with "how much" the particles move.
- We can say that the entropy is associated with "the way" the particles moves.
- Entropy according to Boltzmann: $S = K_B \log W$
  W is the number of configurations

Second Principle: "In a spontaneous transformation the entropy always increases"
Is equivalent to say that "If a system can be in a number of different states, all compatible with the conservation laws, then it will evolve in order to attain the equilibrium condition identified with the **most probable state** among all the possible states it can be in. "

Friction: there is a connection between the process of dissipating energy (a typical irreversible, i.e. non-equilibrium process) and the process of fluctuating at equilibrium with the gas: the Fluctuation-Dissipation Theorem.

To learn more:

Energy Management at the Nanoscale
L. Gammaitoni

in the book "ICT - Energy - Concepts Towards Zero - Power Information and Communication Technology" InTech, February 2, 2014